

Semester Two Examination, 2022

Question/Answer booklet

MATHEMATICS  
APPLICATIONS  
UNITS 3&4

**SOLUTIONS**

Section One:  
Calculator-free

|  |  |  |  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- | --- | --- | --- |
| WA student number: In figures |  |  |  |  |  |  |  |  |  |  |

In words

Your name

|  |  |
| --- | --- |
| Number of additional answer booklets used (if applicable): |  |

## Time allowed for this section

Reading time before commencing work: five minutes

Working time: fifty minutes

## Materials required/recommended for this section

***To be provided by the supervisor***

This Question/Answer booklet

Formula sheet

***To be provided by the candidate***

Standard items: pens (blue/black preferred), pencils (including coloured), sharpener,  
correction fluid/tape, eraser, ruler, highlighters

Special items: nil

## Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

## Structure of this paper

|  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- |
| Section | Number of questions available | Number of questions to be answered | Working time (minutes) | Marks available | Percentage of examination |
| Section One: Calculator-free | 7 | 7 | 50 | 51 | 35 |
| Section Two: Calculator-assumed | 12 | 12 | 100 | 99 | 65 |
|  | | |  | **Total** | 100 |

## Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet preferably using a blue/black pen.  
Do not use erasable or gel pens.

3. You must be careful to confine your answers to the specific question asked and to follow any instructions that are specific to a particular question.

4. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

5. It is recommended that you do not use pencil, except in diagrams.

6. Supplementary pages for planning/continuing your answers to questions are provided at the end of this Question/Answer booklet. If you use these pages to continue an answer, indicate at the original answer where the answer is continued, i.e. give the page number.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

Section One: Calculator-free 35% (51 Marks)

This section has**seven** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 50 minutes.

Question 1 (5 marks)

The daily number of gas bottle refills () over a period of three weeks at a service station is shown in the plot below. The Monday of Week 1 corresponds to .

<EFOFEX>
id:fxd{978aa638-55b9-4ef1-b0a1-b6cdd7aa2c53}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ correctly plots at least three points  ü correctly plots all points and joins them |

The number of refills during Week 4 are shown in the table below.

|  |  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- | --- |
| Day | Mon | Tue | Wed | Thu | Fri | Sat | Sun |
| Time, |  |  |  |  |  |  |  |
| Refills, |  |  |  |  |  |  |  |

(a) Complete the time series plot by adding the above data for Week 4. (2 marks)

(b) Describe the trend, seasonality and any other features of the time series plot. (3 marks)

|  |
| --- |
| Solution |
| The trend of the plot is increasing.  The seasonality has a weekly cycle, as evidenced by the highest number of refills every Sunday and the lowest every Friday, except for a possible outlier on day , when refills appear to be unusually low. |
| Specific behaviours |
| ✓ states the overall trend  ü refers to length, high and low parts of season  ü refers to likely outlier on day |

Question 2 (6 marks)

(a) A farmer has four fields that are to be used to grow crops of swedes, wheat, carrots and potatoes. Field is suitable for swedes or carrots, Field for swedes or potatoes, Field for wheat or carrots and Field for potatoes.

(i) Represent this information using a bipartite graph. (2 marks)

|  |
| --- |
| Solution |
| <EFOFEX> id:fxd{7d9d0077-5a9c-4ba6-bfb8-1d8be264b9d0}  FXData:  </EFOFEX> |
| Specific behaviours |
| ✓ two labelled sets of vertices  ü correct bipartite graph |

(ii) Given that all four crops must be grown at the same time, and only one crop can be grown per field, in which field should the farmer grow the crop of carrots? Justify your answer. (1 mark)

|  |
| --- |
| Solution |
| Field . Although fields and are both suitable for carrots, must be used for wheat, leaving for carrots. |
| Specific behaviours |
| ✓ correct field, with reasonable justification |

<EFOFEX>
id:fxd{1558546b-8b22-4d32-8e12-a37b6f8563c3}

FXData:
</EFOFEX>(b) The graph shown to the right represents  
unsealed tracks between seven barns on  
the farm.

(i) State the length of the longest cycle in the graph. (1 mark)

|  |
| --- |
| Solution |
| Length is . |
| Specific behaviours |
| ✓ correct length |

(ii) Explain why the graph is not Hamiltonian. (1 mark)

|  |
| --- |
| Solution |
| It does not contain a cycle through all seven vertices. |
| Specific behaviours |
| ✓ reasonable explanation |

(iii) An edge can be added to the graph so that it becomes Hamiltonian. State a pair of vertices between which such an edge should be added. (1 mark)

|  |
| --- |
| Solution |
| Between and either or . |
| Specific behaviours |
| ✓ correct pair of vertices |

Question 3 (7 marks)

An employee at a reticulation company represented three lawn watering systems using  
graphs , and .

(a) Graph has adjacency matrix .  
  
Draw graph in the plane, labelling the vertices and . (3 marks)

|  |
| --- |
| Solution |
| <EFOFEX> id:fxd{be34a3a8-fa96-40fa-be76-81e2cf39a688}  FXData:  </EFOFEX> |
| Specific behaviours |
| ✓ graph with five vertices, correctly labelled  ü loop at vertex  ü correct graph drawn in the plane (no crossed edges) |

(b) Graph is a connected planar graph with edges and faces.  
Determine the number of vertices in graph . (2 marks)

|  |
| --- |
| Solution |
| Using Euler's formula .  Hence and so there are vertices in graph . |
| Specific behaviours |
| ✓ correctly substitutes into Euler's formula  ü correct number of vertices |

(c) Graph has vertices, is a tree, and the longest trail it contains has length .  
Draw graph . (2 marks)

|  |
| --- |
| Solution |
| <EFOFEX> id:fxd{d5986a76-00c5-4061-817b-c979136d1043}  FXData:  </EFOFEX> |
| Specific behaviours |
| ✓ tree with vertices  ü correct graph |

Question 4 (6 marks)

A computer is running some image recognition software that is learning a new task.

On the run of the software, the time taken to complete the task, milliseconds, is given by the recurrence relation

(a) Use the recurrence relation to complete the table below. (2 marks)

|  |  |  |  |  |  |  |
| --- | --- | --- | --- | --- | --- | --- |
|  |  |  |  |  |  |  |
|  |  |  |  |  | **7** |  |

|  |
| --- |
| Solution |
| See table |
| Specific behaviours |
| ✓ at least correct entries  ü all correct entries |

(b) Plot the times on the axes below. (2 marks)

<EFOFEX>
id:fxd{284dd110-d6b3-4a90-88f0-bf27bb5a8ab9}

FXData:

</EFOFEX>

|  |
| --- |
| Solution |
| See graph |
| Specific behaviours |
| ✓ at least 3 correct points  ü all correct points, no connecting lines |

(c) According to the recurrence relation, the time taken to complete the task will never be quicker than milliseconds. Determine, with justification, the value of . (2 marks)

|  |  |
| --- | --- |
| Solution | |
| In steady state, .  Substituting into recurrence relation: | When then  Hence steady state when . |
| Specific behaviours | |
| ✓ any reasonable justification  ü correct value for | |

Question 5 (8 marks)

Four workers, P, Q, R and S, are each to be assigned to one of four tasks, 1, 2, 3 and 4. Each worker must be assigned to one task, and each task must be done by exactly one worker. Worker R cannot be assigned to task 2. The profit, in dollars, that each worker would generate when assigned to each task is shown in the table below.

|  |  |  |  |  |
| --- | --- | --- | --- | --- |
|  | **1** | **2** | **3** | **4** |
| **P** |  |  |  |  |
| **Q** |  |  |  |  |
| **R** |  |  |  |  |
| **S** |  |  |  |  |

(a) Explain two initial modifications that must be made to the table so that the Hungarian algorithm may be applied to the resulting figures in order to determine the maximum total profit that can be generated by the four workers. (2 marks)

|  |
| --- |
| Solution |
| Set the profit for worker R, task 2, to any value less than , such as .  Subtract each entry from (or larger) |
| Specific behaviours |
| ✓ indicates need for, or states, a suitable dummy value for R2  ü indicates need to convert from maximising to minimising problem |

(b) Modify the table as required in part (a). (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ suitable value for C3 and subtracts from constant to leave all non-negative values |

(c) Reducing rows first, use the Hungarian algorithm to determine the maximum total profit and the assignment of workers required to achieve this maximum. (5 marks)

|  |
| --- |
| Solution |
| Reduce rows, columns and then apply algorithm to create extra zeros:  (NB Zeros in middle table can be covered with lines: rows and , and column , leaving smallest uncovered number ).  Required assignment (bolded) is .  This assignment generates a maximum profit of . |
| Specific behaviours |
| ü correctly reduces rows  ü correctly reduces columns  ü correctly creates extra zeros  ü states assignment  ü states maximum profit |

Question 6 (11 marks)

(a) The tasks required to complete a project are shown in the table below, together with the duration of each task in days, and their immediate predecessor(s).

|  |  |  |
| --- | --- | --- |
| Task | Duration (days) | Immediate predecessor(s) |
| A |  | None |
| B |  | None |
| C |  | A |
| D |  | B |
| E |  | B |
| F |  | C |
| G |  | C |
| H |  | D |
| J |  | D, E |
| K |  | D, E |
| L |  | G, H, J |

(i) Construct a project network to represent the information in the table. (4 marks)

|  |
| --- |
| Solution |
| <EFOFEX> id:fxd{9513a58a-8147-4078-a91b-8b533b33451f}  FXData:  </EFOFEX> |
| Specific behaviours |
| ✓ correctly add tasks A, B, C, D, E  ü correctly adds remaining tasks  ü uses dummy edge to show D must finish before J and K can start  ü indicates direction and duration of all tasks |

(ii) State the earliest time that task G can commence. (1 mark)

|  |
| --- |
| Solution |
| The earliest start time is days. |
| Specific behaviours |
| ✓ correct time |

(b) The network showing the tasks that need to be undertaken to complete a different project is shown below. The duration of each task, in minutes, is shown in brackets.

<EFOFEX>
id:fxd{41b68a40-47f4-445f-ac02-8998782321a5}

FXData:

</EFOFEX>

(i) What does the dotted line on the network indicate? (1 mark)

|  |
| --- |
| Solution |
| Task D cannot start until task B has finished. |
| Specific behaviours |
| ✓ states correct interpretation |

(ii) Determine, in order, the tasks that lie on the critical path and the minimum completion time for the project. (2 marks)

|  |
| --- |
| Solution |
| Tasks on critical path are B, D, J, and the minimum completion time is minutes. |
| Specific behaviours |
| ✓ correctly lists task on critical path  ü correctly states the minimum completion time |

(iii) Determine the float time for task H. (1 mark)

|  |
| --- |
| Solution |
|  |
| Specific behaviours |
| ✓ correct float time |

(iv) If task B could be completed in only minutes, how would this affect the critical path and minimum completion time? (2 marks)

|  |
| --- |
| Solution |
| The tasks on critical path would change to C, G, L, M, and the minimum completion time would be minutes. |
| Specific behaviours |
| ✓ correctly lists tasks on new critical path  ü correctly states the new minimum completion time |

Question 7 (8 marks)

The edges in the graph below represent footpaths between buildings (shown as vertices) on the extensive campus of a research centre. Each edge weight represents the time, in minutes, that a supervisor takes to walk along that footpath to check that it is clean and in good condition.

<EFOFEX>
id:fxd{d6cf2e80-723d-4556-a842-5fac2dbcf404}

FXData:

</EFOFEX>

(a) State, with justification, whether the graph is Eulerian, semi-Eulerian or neither. (2 marks)

|  |
| --- |
| Solution |
| The graph has exactly two odd vertices and so it is semi-Eulerian. |
| Specific behaviours |
| ✓ states semi-Eulerian, with justification  ü justification indicating graph has **exactly** two odd vertices and the rest are even  And the rest |

The supervisor's office is in building . The least time that the supervisor takes to complete an inspection of all footpaths and return to their office is minutes.

(b) Determine the value of , the time to walk along the footpath from to . (4 marks)

|  |
| --- |
| Solution |
| Semi-Eulerian trail starts at , ends at . Then return to along .  Sum of edges:  Hence, with repeated edge , we get . |
| Specific behaviours |
| ü reasonable attempt to find length of semi-Eulerian trail  ü correct simplified length of semi-Eulerian trail  ü forms equation using semi-Eulerian trail length plus repeated edge  ü correct value of |

(c) The supervisor is in their office. At am, they are asked to inspect the footpath between buildings and . Determine the earliest time that they could finish the inspection of this footpath if they left their office immediately. (2 marks)

|  |
| --- |
| Solution |
| Shortest path to reach one end of edge is minutes.  Hence finish inspection minutes after leaving office, at am. |
| Specific behaviours |
| ✓ indicates shortest path to reach one end of edge  ü correct finish time |

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

Supplementary page

Question number: \_\_\_\_\_\_\_\_\_

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